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Data Mining of ESEL and JET Probe Data

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Data Mining of
ESEL & JET probe data

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Motivation

- Fusion devices: Turbulence
- Internal plasma dynamic: Statistical properties
- Link: plasma turbulence and K41 model ?
- Universal properties of edge magnetized plasmas: Fluctuations are self-similar in space & time

Motivation

- Fusion devices: Turbulence
- Internal plasma dynamic: Statistical properties
- Link: plasma turbulence and K41 model ?
- Universal properties of edge magnetized plasmas: Fluctuations are self-similar in space & time due to intermittent structures
  - Memory effects
  - Large-scale correlation (anomalous diffusion)
- coherent structures (vortices, zonal flows, streamers, blobs)

Contents

- Introduction: SOC & Self-Similarity
- Methods to determine Hurst Coefficient
- JET: Hurst vs. ρ
- ESEL: Hurst vs. ρ

Introduction: Self-Organized Criticality (SOC)

- Phenomena like: self-organization of NLDS + near-critical
  length scale : fractal geometry
  time scale : power spectra, behavior $1/f$
- SOC systems = scale invariant in space and time
- Experiments: intermittency and self-similar flucutations
- Memory-effects and long-range correlations (persistence)
- Hurst parameter: suitable measure of persistence in time series

Statistical sense : $x(bt) \quad "=" \quad b^H x(t) \qquad \forall\ b, t > 0$

Introduction Self-Similarity

Statistical sense : $x(bt) \quad "=" \quad b^H x(t) \qquad \forall\ b, t > 0$

Hurst exponent :

$H = 0.0$ White noise

$H < 0.5$ anti-correlation(restriction of the noise process)

$H = 0.5$ Brownian noise

$H > 0.5$ persistence, fractal Browian process

$H = 1.0$ exactly self-similar process

fGn & fBm

stationary process fractional Gaussian noise (fGn)
autocorrelation coefficient

$$\rho_n(X_t, X_{t+n}) = 0.5 \cdot \left(|n+1|^{2H} - 2|n|^{2H} + |n-1|^{2H}\right)$$

nonstationary process fractal Browian motion (fBm)
cumulative sum of a fGn process $Y_t = \sum_{t_0}^{t} X_i$

The processes do <u>not</u> share the same correlation structure!

Methods - Overview

- Rescaled range (R/S) method, time lag $> 5 - 10$ local autocorrelation time
- Fourier spectra (just monofractal)
- Structurfunctions, time lag > 1 local autocorrelation time
- Scaled window variance method

Methods - Power spectral density (PSD)

Power Law Scaling, Wiener Khinchine Theorem link H with β

$$\lim_{\omega \to \infty} S(\omega) \sim \omega^{-\beta}$$

stationary process	$1 < \beta = 2H + 1 < 3$
nonstationary process	$-1 < \beta = 2H - 1 < 1$ (time average PSD)

Note: $H = H(q = 2)$ thus only monofractal analysis

Scaled Windows Variance Method (SWVM) for fBm

$$V(m) = \frac{m}{N} \sum^{N/m} \sigma_i \sim m^H$$

Note: $H = H(q = 1)$ thus only monofractal analysis

Example

K41 Theory each point of the $\vec{v}$-field: $\Delta v_L \sim L^{1/3}$
energy spectrum, no viscosity, $E_k \sim k^{-5/3}$

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Structure Functions (SF) for fBm

$$S_q(\Delta t) \; = < |x(t+\Delta t) - x(t)|^q > \sim \; \Delta t^{\xi(q)}$$

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Integrated Structure Functions (intSF) for fGn

$W_t = \sum^t X_i$

$$intS_q(\Delta t) \; = < |W(t+\Delta t) - W(t)|^q > \sim \; \Delta t^{\xi(q)}$$

Measure of self-similarity Hurst $H = E + 1 - D = \xi(q)/q$

Monofractal

$$\xi(q) = Hq$$

Multifractal

$$\xi(q) = H(q)q$$

Methods - Multi-/Monofractal

Measure of self-similarity Hurst $H = E + 1 - D = \xi(q)/q$

Monofractal

$$\xi(q) = Hq$$

Multifractal

$$\xi(q) = H(q)q$$

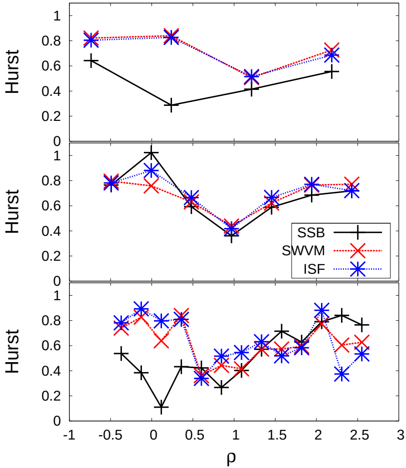
Intermittent

She and Leveque (SL) predicting scaling model

$$\xi(q) = \frac{q}{9} + 2\left(1 - (\frac{2}{3})^{q/3}\right)$$

- Example: ESEL, structure functions, integrated structure functions (gnu/den_sat3x3_temperature_data.gnu)
- Hurst vs. radial direction (gnu/den_sat3x3_temperature_all_H.gnu)
- $\xi(q)$ vs. q (gnu/den_sat3x3_temperature_anstiege.gnu)

- Langmuir-probe measurement: ion saturation current $j_{\text{sat}} \propto n \cdot \sqrt{T_e}$.
- density fluctuation behavior is dominant
- Windows size $\Delta\rho = [0.25, 0.5, 1.0]$



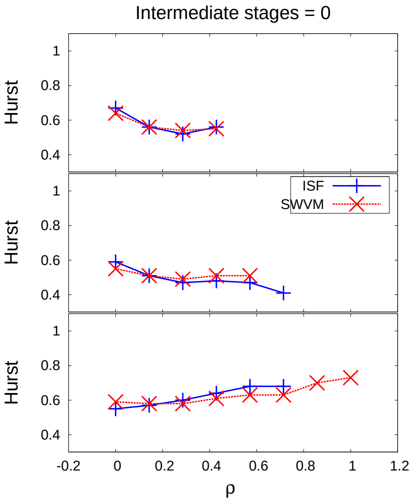
Electrostatic interchange model

$$
\begin{aligned}
\frac{\mathrm{D}}{\mathrm{D}t} n &= \Lambda_n n - n\mathcal{C}(\phi) + \mathcal{C}(n T_e) \\
\frac{\mathrm{D}}{\mathrm{D}t} T_e &= \Lambda_{T_e} T_e - \frac{2}{3} T_e \mathcal{C}(\phi) + \frac{7}{3} T_e \mathcal{C}(T_e) + \frac{2}{3}\frac{T_e^2}{n}\mathcal{C}(n) \\
\frac{\mathrm{D}}{\mathrm{D}t} \Omega &= \Lambda_\Omega \Omega + \mathcal{C}(n T_e)
\end{aligned}
$$

with

$$
\mathcal{C}(.) = -\frac{\rho_{s,0}}{R_0}\frac{\partial}{\partial y}. \quad , \; \Lambda_. = D_{\perp .}\nabla_\perp^2 . - \frac{.}{\tau_{\parallel .}} \quad , \; \frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + v_E \cdot \nabla\phi
$$

ESEL - Probe Movement



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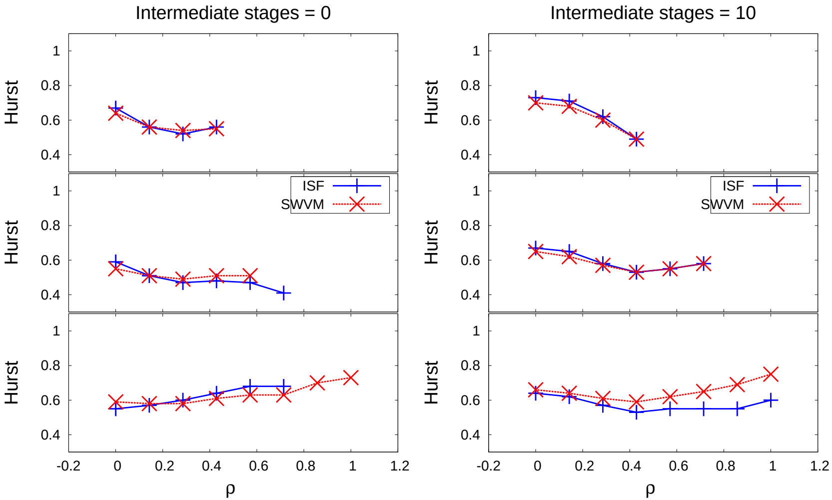

Conclusion

- Confined plasma: Persistence dominate the dynamics, transport processes
- SOL: Random processes dominate the dynamics $H \approx 0.5$
- Wall Shadow: High Hurst coefficient due to low statistics

$$\begin{array}{lcl} \textbf{Experiment} & : & H \approx 0.8 \\ \textbf{Simulation} & : & H \approx 0.7 \end{array}$$

Outlook

- Hurst coefficient in dependence of the shear flow
- Which parameter Multifractality in the simulation?
- Persistence of potential fluctuations are different from j_{sat}?
- Find jet shots with ESS to study turbulent models ...
- Is a 3D code more suitable?

Thanks for your attention!